PHYSICAL REVIEW E, VOLUME 64, 061402

Thermophoretic and ponderomotive forces in a linear cluster of particles

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(Received 10 January 2001; published 21 November 2001)

We investigate amplification of thermophoretic and ponderomotive forces caused by renormalization of static fields in linear clusters of particles with scale separation. We found analytically the dependence of the forces acting on the particles in a cluster as a function of the number of particles in a cluster and material characteristics of the particles and the surrounding fluid. We analytically determined the velocity of stationary motion of particles, velocity distribution in a surrounding viscous fluid, and the thermophoretic force when the particles remain stationary due to the applied constraint forces.

DOI: 10.1103/PhysRevE.64.061402 PACS number(s): 47.15.Pn, 83.10.Pp

I. INTRODUCTION

Behavior of particles in liquid or gaseous media in the presence of external fields has many facets and is of interest in various applications (see, e.g., [1] and reference therein). One of these problems, namely, the dynamics of the ensemble of particles in a viscous fluid, has a long history (see, e.g., [2–4] and references therein). The principal difficulty in solving this problem is associated with the necessity to account for the long range many body hydrodynamic interaction among particles. This problem still remains a challenge not only to analytical but also approximate and numerical methods [5]. In this study, we considered a linear cluster of particles whereby the size of each particle is strongly different from the size of all other particles. In this particular case, the analytical solution can be derived since one has to take into account only the influence of a larger particle on the adjacent smaller particle in a cluster while the opposite effect can be neglected. The considered system is not only of academic interest but also of technological interest since as showed below that it can be used for extraction of small particles from a suspension.

We investigated the motion of a cluster of particles in a viscous host medium with an imposed external temperature gradient. Thermophoresis, i.e., motion of particles due to temperature gradient in a host medium, is of great significance in technology and various naturally occurring phenomena (see Ref. [6] and references therein). Thus, e.g., thermophoretic scavenging of aerosols by cloud droplets plays an important role in pollutants transport. The thermophoretic interaction of particles is relevant in studies of nonlinear optical effects in ferrofluids [7] and thermomagnetophoretic transport in ferrofluid colloids [8]. Spatially ordered structures in the arrangement of small liquid droplets or solid particles embedded in a thermal plasma were observed in several experiments (see, e.g., [9] and references therein). Recently it was suggested that thermophoretic forces may play a crucial role in the formation of these structures [10].

In order to define the problem more accurately, let us consider a cluster consisting of N spherical particles located at the line in the imposed external field $\vec{A}$. This field can be electrostatic field or magnetostatic field, electric current with density $\vec{j}$, temperature gradient, etc. It is assumed that coefficient of response of the particle to the corresponding field κ_1 (e.g., coefficient of thermal conductivity, dielectric permittivity, etc.) is essentially different from the response coefficient of the host medium κ_0 so that $\alpha=\kappa_0/\kappa_1\neq 1$. We will assume also that the sizes of the particles $R_1,R_2,\ldots,R_N$ satisfy the condition $R_1\gg R_2\gg\ldots\gg R_N$ and that the distance between the adjacent particles is much larger than the size of the smaller particle and less than the size of the larger particle. Under these conditions, there occurs strong amplification of the static field in small spatial scales [11]. Thus, the static field in the vicinity of a small particle with a radius R_k is by the factor λ^k stronger than the applied external field where the coefficient λ depends upon the orientation of the linear cluster with respect to the direction of the external field and upon the parameter α. Taking into account this amplification, we determined the velocity distribution in a viscous fluid containing N particles, with velocities of the particles and thermophoretic forces acting upon the particles.

This paper is organized as follows. In Sec. II we describe the general scheme for calculating the static field in a linear cluster of particles. In Sec. III assuming that the field calculated in Sec. II corresponds to the electrostatic field, we calculated thermodynamic pressure inside the kth liquid particle. A coefficient α in this case is defined as $\alpha=\varepsilon_0/\varepsilon_1$, where ε_0 and ε_1 are permittivities of the host medium and liquid particles, respectively. It is demonstrated that if the coefficient α satisfies the condition $\alpha\ll 1$, the thermodynamic pressure inside small liquid particles is considerably less than the pressure inside the larger particles in the cluster. In Sec. IV assuming that the calculated field describes the distribution of a temperature gradient inside a system subjected to the external temperature gradient $\vec{A}$, we calculated the velocities of the stationary motion of particles $\vec{V}_1,\vec{V}_2,\ldots\vec{V}_N$. In the same section, we also considered another situation in which the particles remain stationary in spite of the applied temperature gradient due to the applied constraint forces. In the latter case, we calculated the thermophoretic forces acting on the particle, which determine the constraint forces that

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must be applied to each particle in order to keep it stationary in the field of the external temperature gradient.

II. MATHEMATICAL MODEL

Consider a system comprising spherical particles with radii $R_i, i=1,...,N$ with coefficient of response to the static field κ_1 immersed into the host medium with response coefficient κ_0 that differs from κ_1. The system is subjected to the external potential field φ that is determined by the equation

$$\vec{\nabla}^2 \varphi = 0 \tag{1}$$

with boundary conditions $\vec{\nabla}\varphi = \vec{A}$ as $|\vec{r}-\vec{c}_i| \to \infty$, where $\vec{c}_i$ is radius vector of the center of the ith particle. The potential φ is continuous and the flux of the field $\kappa\vec{\nabla}\varphi$ at the surface of the ith particle satisfies the condition

$$\vec{n}_i \cdot [\kappa \vec{\nabla} \varphi]_i = 0, \tag{2}$$

where $\vec{n}_i$ is a unit external normal vector at the surface of the ith particle, $[\beta]_i = \beta_i^+ - \beta_i^-$ and $\beta_i^\pm$ are the magnitudes of β at the external and internal surfaces, respectively.

Assume that the radii of the particles R_i and the distances between the surfaces of the particles $d_{i,i+1}$ satisfy the following condition:

$$R_i \gg d_{i,i+1} \gg R_{i+1}. \tag{3}$$

Assume also that the external field is homogeneous $\vec{A} = \mathrm{const}$, and represent the potential φ as follows:

$$\varphi = \vec{A} \cdot \vec{r} + \sum_{i=1}^{N} \tilde{\varphi}_i(\vec{r}-\vec{c}_i). \tag{4}$$

The boundary value problem (1)–(2) can be solved by determining $\tilde{\varphi}_1, \tilde{\varphi}_2, \ldots, \tilde{\varphi}_N$ sequentially. To this end, let us note that the distortion of the field produced by the ith particle outside the particle is $\sim R_i^3/|\vec{r}-\vec{c}_i|^3$, and condition (3) implies that in determining potential $\tilde{\varphi}_{i-1}$ we can neglect the effect of the ith particle. Then condition (2) yields the following boundary condition for the potential $\tilde{\varphi}_k$:

$$\vec{n}_k \cdot [\kappa \vec{\nabla} \tilde{\varphi}]_k = (\kappa_1 - \kappa_0)\vec{n}_k \cdot \sum_{i=1}^{k-1} \vec{\nabla}\tilde{\varphi}_i + (\kappa_1 - \kappa_0)(\vec{n}_k \cdot \vec{A}) \tag{5}$$

and $\tilde{\varphi}_k \to 0$ as $|\vec{r}-\vec{c}_k| \to \infty$.

Now let us solve Eqs. (1), (4), (5). The potential $\tilde{\varphi}_1$ is known (see, e.g., Ref. [12], Chap. 5, Sec. 50):

$$\tilde{\varphi}_1 = \xi\left[\theta_-(x_1) + \theta_+(x_1)\frac{R_1^3}{|\vec{r}-\vec{c}_1|^3}\right]\vec{A}\cdot(\vec{r}-\vec{c}_1), \qquad \xi = \frac{\kappa_0 - \kappa_1}{\kappa_1 + 2\kappa_0}, \tag{6}$$

where $\theta_\pm(z) \equiv \theta(\pm z)$, $\theta(x)$ is Heaviside function and $x_1 = |\vec{r}-\vec{c}_1| - R_1$. Now we can determine the potential $\tilde{\varphi}_2$. According to Eq. (5), we must determine potential $\tilde{\varphi}_1$ at the surface of the second particle $|\vec{r}-\vec{c}_2| = R_2$. To this end, let us represent the potential $\tilde{\varphi}_1$ with respect to the center of the second particle $\vec{c}_2$. In the leading order in the vicinity of the surface $|\vec{r}-\vec{c}_2| - R_2 \ll |\vec{c}_2 - \vec{c}_1|$, the potential $\tilde{\varphi}_1$ can be written as

$$\tilde{\varphi}_1 = y_1^3 \xi(\vec{r}-\vec{c}_2)\cdot[\vec{A} - 3(\vec{A}\cdot\vec{S}_{12})\vec{S}_{12}], \quad \vec{S}_{12} = \frac{\vec{c}_1 - \vec{c}_2}{|\vec{c}_1 - \vec{c}_2|}, \qquad y_1 = \frac{R_1}{|\vec{c}_1 - \vec{c}_2|}. \tag{7}$$

Hereafter we will consider the case when the centers of all particles are located on the straight line. Let us analyze two of the most simple cases, namely, when the particles are aligned in the direction of the field and $\vec{S}_{12} = \pm\vec{A}/|\vec{A}|$, and, the second one, when $\vec{S}_{12}\cdot\vec{A} = 0$. Note that condition (3) implies that $y_i^3 = R_i^3/|\vec{c}_i - \vec{c}_{i+1}| \cong 1$. In the first case, using the boundary condition (5), we obtain an expression similar to Eq. (6):

$$\tilde{\varphi}_2 = \xi\left[\theta_-(x_2) + \theta_+(x_2)\frac{R_2^3}{|\vec{r}-\vec{c}_2|^3}\right]\vec{A}_1\cdot(\vec{r}-\vec{c}_2), \tag{8}$$

where $\vec{A}_1 = \vec{A}(1-2\xi) = \vec{A}\,3/(1+2\alpha)$, $x_2 = |\vec{r}-\vec{c}_2| - R_2$, $\alpha = \kappa_0/\kappa_1$.

Similarly, we can determine a potential $\tilde{\varphi}_3$. To this end, we rewrite potentials $\tilde{\varphi}_1$ and $\tilde{\varphi}_2$ in the vicinity $|\vec{r}-\vec{c}_3| \ll |\vec{c}_1 - \vec{c}_3|$ and $|\vec{r}-\vec{c}_3| \ll |\vec{c}_2 - \vec{c}_3|$. Since condition (3) implies that $R_2^3/|\vec{c}_3 - \vec{c}_2|^3 \approx R_1^3/|\vec{c}_1 - \vec{c}_3|^3 \approx 1$, we obtain

$$\tilde{\varphi}_3 = \xi\left[\theta_-(x_3) + \theta_+(x_3)\frac{R_3^3}{|\vec{r}-\vec{c}_3|^3}\right]\left(\frac{3}{1+2\alpha}\right)^2\vec{A}\cdot(\vec{r}-\vec{c}_3),$$

Repeating the above procedure, we find that

$$\tilde{\varphi}_k = \xi\left[\theta_-(x_k) + \theta_+(x_k)\frac{R_k^3}{|\vec{r}-\vec{c}_k|^3}\right]\left(\frac{3}{1+2\alpha}\right)^{k-1}\vec{A}\cdot(\vec{r}-\vec{c}_k). \tag{9}$$

Equations (4) and (9) determine the solution of the boundary value problem (1)-(2). Using these equations, we obtain the following formula for the strength of the field $\vec{E} = -\vec{\nabla}\varphi$:

$$\vec{E} = \vec{A} + \vec{A}\xi\sum_{i=1}^{N}\left[\theta_-(x_i) + \theta_+(x_i)\frac{R_i^3}{|\vec{r}-\vec{c}_i|^3}\right]\left(\frac{3}{1+2\alpha}\right)^{i-1} - \xi\sum_{i=1}^{N}\theta_+(x_i)\frac{R_i^3}{|\vec{r}-\vec{c}_i|^3}\left(\frac{3}{1+2\alpha}\right)^{i-1} 3\vec{n}_i(\vec{A}\cdot\vec{n}_i). \tag{10}$$

The characteristic feature of Eq. (10) is that due to condition (3) the contributions of various terms change by the order of magnitude depending on the location. In order to calculate the field using Eq. (10) at the arbitrary point in space without exceeding the accuracy, one must neglect the small terms according to condition (3). Let us determine the field $\vec{E}$ in the domain $|\vec{r}-\vec{c}_k|\sim R_k$. In the region $x_k=|\vec{r}-\vec{c}_k|-R_k<0$, Eq. (10) yields

$$\vec{E}_k^{\text{in}}=\alpha\left(\frac{3}{1+2\alpha}\right)^k\vec{A}, \tag{11}$$

and in the region $x_k=|\vec{r}-\vec{c}_k|-R_k>0$

$$\vec{E}_k^{\text{out}}=\left(\frac{3}{1+2\alpha}\right)^{k-1}\left[\vec{A}+\xi\frac{R_k^3}{|\vec{r}-\vec{c}_k|^3}[\vec{A}-3\vec{n}_k(\vec{A}\cdot\vec{n}_k)]\right]. \tag{12}$$

Equation (12) implies that the normal component of the strength of the field at the external surface of the kth particle,

$$E_{k,n}^{\text{out}}=\left(\frac{3}{1+2\alpha}\right)^k A_n, \tag{13}$$

increases by a factor 3^k when $\alpha\ll 1$.

Similarly, one can solve a problem when the particles are aligned normally to the direction of the field. In this case for the strength of the field inside the kth particle instead of Eq. (11) we obtain

$$\vec{E}_k^{\text{in}}=\left(\frac{3\alpha}{1+2\alpha}\right)^k\vec{A}, \tag{14}$$

and for the field outside the kth particle instead of Eq. (12), we obtain

$$\vec{E}_k^{\text{out}}=\left(\frac{3\alpha}{1+2\alpha}\right)^{k-1}\left[\vec{A}+\xi\frac{R_k^3}{|\vec{r}-\vec{c}_k|^3}[\vec{A}-3\vec{n}_k(\vec{A}\cdot\vec{n}_k)]\right]. \tag{15}$$

The normal component of the strength of the field at the external surface of the kth particle is

$$E_{k,n}^{\text{out}}=\frac{3}{1+2\alpha}\left(\frac{3\alpha}{1+2\alpha}\right)^{k-1}A_n$$

and the tangential component

$$E_{k,\tau}^{\text{out}}=E_{k,\tau}^{\text{in}}=\left(\frac{3\alpha}{1+2\alpha}\right)^k A_\tau.$$

III. RENORMALIZATION OF THE THERMODYNAMIC PRESSURE IN PARTICLES IN THE LINEAR CLUSTER IN THE EXTERNAL ELECTRIC FIELD

Using the above results, let us determine ponderomotive forces acting on the particle in the cluster in electric field $\vec{E}$ when a particle is homogeneous and the charge of the particle is zero. If the total energy of the electric field,

$$W=\int\frac{\varepsilon\vec{E}^2}{8\pi}dV, \tag{16}$$

depends upon the coordinate of the center of mass of a particle $\vec{c}_i$, then the ponderomotive force with a magnitude

$$\vec{F}_i=-\frac{\partial W}{\partial\vec{c}_i}$$

acts on the particle and causes its translational motion. If the integral in Eq. (16) can be represented as a sum of integrals that are invariant with respect to the transformation $\vec{r}\to\vec{r}-\vec{c}_i$, then the ponderomotive forces cannot cause the translational motion of the particle. It is exactly the situation which occurs in the considered configuration in the framework of the adopted accuracy. However, already in this approximation the particle is subjected to compressive or tensile stresses that depend upon the location of the particle inside a cluster, and in this case the main effect is the renormalization of a thermodynamic pressure. In principle, the thermodynamic pressure can be determined by using Eq. (16) and variational principles of thermodynamics (for details see Ref. [11]). However, in this study we will determine pressure inside the kth particle by using the formula for a stress tensor and condition of mechanical equilibrium (see Ref. [13], Chap. 2, Sec. 15). The expression for the stress tensor reads

$$\sigma_{ik}=-\left[p_0(\rho,T)+\frac{\varepsilon\vec{E}^2}{8\pi}\right]\delta_{ik}+\frac{\varepsilon E_iE_k}{4\pi}.$$

Condition for continuity of the normal component of the stress tensor at the surface of the particle $[\sigma_{nn}]_k=0$ yields

$$\Delta p_k=D_{k,n}(E_{k,n}^{\text{in}}-E_{k,n}^{\text{out}})+\frac{E_{k,\tau}^2}{8\pi}(\varepsilon_0-\varepsilon_1), \tag{17}$$

where $\vec{D}=\varepsilon\vec{E}$ is an electric induction, $\Delta p_k=p_{\text{in}}-p_{\text{out}}$ is a difference between the thermodynamic pressures inside the kth particle and in the host medium in the vicinity of the particle. Substituting the obtained expressions given above, for the normal and tangential components of the strength of the electric field E_n and E_τ into Eq. (17), we obtain

$$\Delta p_k=\frac{9\xi_\varepsilon\varepsilon_0E_0^2(\lambda^2)^{k-1}}{8\pi}\left(\frac{1}{1+2\alpha_\varepsilon}\cos^2\theta+\frac{\alpha_\varepsilon}{1+2\alpha_\varepsilon}\sin^2\theta\right). \tag{18}$$

Here E_0 is the strength of the external field, $\cos\theta=(\vec{n}\cdot\vec{E}_0)/|\vec{E}_0|$, $\alpha_\varepsilon=\varepsilon_0/\varepsilon_1$, $\xi_\varepsilon=(\alpha_\varepsilon-1)/(1+2\alpha_\varepsilon)$, and λ depends upon the geometry. When the linear cluster is aligned with the external field, $\lambda=\lambda_\parallel$, and when it is normal to the direction of external field, $\lambda=\lambda_\perp$,

$$\lambda_\parallel=\frac{3}{1+2\alpha_\varepsilon},\quad \lambda_\perp=\frac{3\alpha_\varepsilon}{1+2\alpha_\varepsilon}. \tag{19}$$

Define thermodynamic pressure averaged over the particle's volume as

$$\langle\Delta p_k\rangle_V=\frac{1}{V}\int\Delta p_k\frac{d^3\vec{r}}{(2\pi)^3},$$

then Eq. (18) yields

$$\langle \Delta p_k \rangle_V = \frac{3\xi \varepsilon_0 E_0^2}{8\pi} (\lambda^2)^{k-1}.$$

The latter equation was derived in Ref. [11] using a different approach. In Ref. [11] it was showed also that renormalization of the thermodynamic pressure results in substantial changes in the dynamics of phase transitions of the first kind in linear clusters of particles in the presence of electric field.

When $\alpha \ll 1$, $\xi \approx -1$, and $\langle \Delta p_k \rangle_V \approx -(9^k/24\pi)\varepsilon_0 E_0^2$, i.e., the pressure in the particle decreases exponentially when its ordinal number in the cluster grows.

IV. THERMOPHORETIC FORCES IN A LINEAR CLUSTER OF PARTICLES

Another kind of fields and forces that are renormalized in the above-described geometry [see Eq. (3)] are thermophoretic forces acting upon macroscopic particles in gases. The origin of these forces was discussed extensively in the literature (see, e.g., Refs. [6,14] and references therein). The mechanism of thermophoretic forces in gases is associated with a temperature gradient that results in the additional momentum flux from gas to particle. Consider a case with relatively large particles of sizes $R_i \gg l$, where l is a free path length of gas molecules. In this study, we seek the solution for times that are much larger than the Stokes time τ_S so that all particles attain their stationary velocities in their motion due to the thermophoretic forces. We will use the system of Stokes equations and neglect convective transfer of thermal energy and buoyancy forces, which can be accounted for without essential changes in the developed approach. The problem will be solved in the zeroth approximation in Knudsen number, $Kn = l/R \ll 1$. Thus, the system of governing equations reads

$$\vec{\nabla} p = \eta \vec{\nabla}^2 \vec{V}, \quad \vec{\nabla} \cdot \vec{V} = 0, \quad \vec{\nabla}^2 T = 0, \tag{20}$$

where p is pressure and η is a kinematic viscosity. The boundary conditions for temperature T correspond to those for potential φ above, and the boundary conditions for velocity $\vec{V}(\vec{r})$ will be presented further. In the stationary regime, the resultant force acting on the particle is $\vec{F} = 0$. Here one has to distinguish between the two cases. The first one is when a particle moves uniformly in the laboratory frame with velocity $\vec{u}_p$. The second case is when a particle is stationary due to the action of the constraint force that compensates the thermophoretic force and $\vec{u}_p = \vec{0}$.

Let us consider the first case. Due to condition (3), we can neglect the effects of small particles in calculating the field of the large particle. Condition (3) also implies that in the vicinity of the kth particle in the scales of the order of the particle's size, the velocity field formed by the preceding $k-1$ particles can be considered as homogeneous. Let the kth particle move with the velocity $\vec{u}_k$. Then, in the frame attached to this particle, the velocity field in the vicinity of the kth particle can be written as follows (see Ref. [12], Chap. 2, Sec. 20):

$$\vec{V}_k(\vec{r}) = \vec{V}_{k\infty} - \vec{u}_k + \vec{\nabla} \times \vec{\nabla} \times f_k(\vec{r}) \vec{A}, \tag{21}$$

where $\vec{A}$ is a temperature gradient far away from the cluster, $\vec{V}_{k\infty}$ is the asymptotic value of the velocity of the gas in the laboratory frame in the region determined by the conditions $|\vec{r} - \vec{c}_k| \gg R_k$, $|\vec{r} - \vec{c}_k| \ll |\vec{c}_k - \vec{c}_{k-1}|$, and

$$f_k(\vec{r}) = a_k |\vec{r} - \vec{c}_k| + \frac{b_k}{|\vec{r} - \vec{c}_k|}, \tag{22}$$

where the coefficients a_k and b_k are determined from the boundary conditions.

In the frame attached to the kth particle, the boundary conditions read

$$V_{k,n} = 0, \quad V_{k,\tau} = \mu(\vec{\nabla} T_k)_\tau, \quad \vec{\nabla} T_0 = \vec{A}, \tag{23}$$

where $V_{k,n}$ and $V_{k,\tau}$ are the normal and tangential components of the gas velocity at the surface of kth particle, respectively. According to the results obtained in Sec. II,

$$(\vec{\nabla} T_k)_\tau = \frac{3\alpha}{1+2\alpha} \lambda^{k-1} A_\tau, \tag{24}$$

where the coefficient λ can assume two values depending on the orientation of the cluster [see Eq. (19)], coefficient α is determined above [see Eq. (8)], and coefficients κ_0 and κ_1 in the formula for α are thermal conductivities of the medium and the particles, respectively. The coefficient of thermal slip μ (see Ref. [14], Chap. 1, Sec. 14) is assumed to be the same for all particles.

We will use Eqs. (19), (24) and as before consider only the cases where a cluster is aligned with vector $\vec{A}$ or normal to it. These assumptions simplify the analysis of the obtained results. The force acting on the kth particle and the pressure in its vicinity are determined by the velocity field [Eqs. (21), (22)] and are given by the following expressions (see [12], Chap. 2, Sec. 20):

$$\vec{F}_k = 8\pi a_k \eta \vec{A}, \quad p_k = p_0 + \eta(\vec{A} \cdot \vec{\nabla})[\vec{\nabla}^2 f_k(\vec{r})]. \tag{25}$$

Since a particle moves with a constant velocity $\vec{F} = 0$, and according to Eqs. (22), (25), $a_k = 0$, $p = p_0$. Taking into account the latter relations, Eqs. (21) can be rewritten as

$$\vec{V}_k(\vec{r}) = \vec{V}_{k\infty} - \vec{u}_k + \frac{b_k}{|\vec{r} - \vec{c}_k|^3} [3\vec{n}_k(\vec{A} \cdot \vec{n}_k) - \vec{A}], \tag{26}$$

where $\vec{n}_k = (\vec{r} - \vec{c}_k)/|\vec{r} - \vec{c}_k|$. In order to determine $\vec{V}_{k\infty}$, let us note that with the required accuracy [see Eq. (3)],

$$\vec{V}_{k\infty} = \vec{V}_{g,k-1}(\vec{r}) \tag{27}$$

for $|\vec{r} - \vec{c}_{k-1}| \approx R_k$, where $\vec{V}_{g,k-1}(\vec{r})$ is a velocity of gas in the vicinity of the $(k-1)$th particle in the laboratory frame.

Let us consider a case where a cluster is aligned with a vector $\vec{A}$. Equation (26) and the above arguments imply that

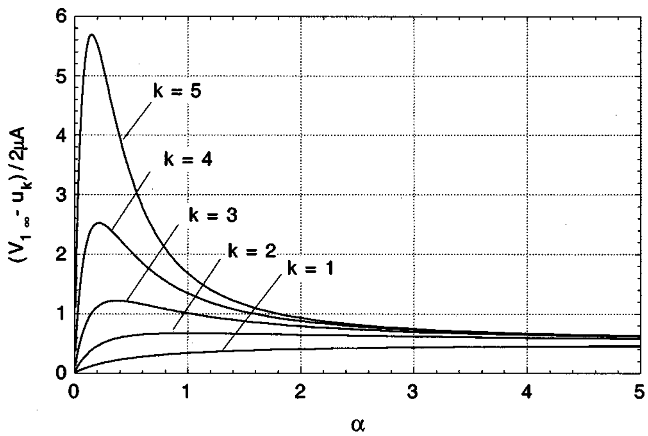

FIG. 1. Dependence of the normalized velocity of the kth particle vs. the ratio of thermal conductivities of the host medium and of the particle $\alpha=\kappa_0/\kappa_1$. Cluster is aligned with the external temperature gradient $\vec{A}$.

$$\vec{V}_{k\infty}=\vec{V}_{k-1,\infty}+\frac{2b_{k-1}}{R_{k-1}^3}\vec{A}. \tag{28}$$

The boundary conditions (23) and Eqs. (24), (26)–(28) yield

$$\vec{u}_k-\vec{V}_{k\infty}=-\frac{2}{3}\mu_k\vec{A}, \quad b_k=-\frac{\mu_k R_k^3}{3}, \tag{29}$$

where

$$\mu_k=\mu\frac{3\alpha}{1+2\alpha}\lambda^{k-1} \quad \text{and} \quad \lambda=\lambda_\parallel=\frac{3}{1+2\alpha}.$$

Solution of the system of Eqs. (28), (29) reads

$$\vec{V}_{k\infty}-\vec{V}_{1\infty}=-\frac{2}{3}\mu\frac{3\alpha}{1+2\alpha}\frac{1-\lambda^{k-1}}{1-\lambda}\vec{A},$$

$$\vec{u}_k-\vec{V}_{1\infty}=-\frac{2}{3}\mu\frac{3\alpha}{1+2\alpha}\frac{1-\lambda^{k}}{1-\lambda}\vec{A}, \tag{30}$$

where $\vec{V}_{1\infty}$ is the velocity of the surrounding fluid far away from the cluster in a laboratory frame. For $\vec{V}_{1\infty}=0$ and $k=1$, Eq. (30) recovers the expression for the thermophoretic velocity of a single particle due to an imposed temperature gradient. The physical meaning of the obtained results is quite transparent. It implies that the velocity of the kth particle is the sum of the velocity due to a local temperature gradient $(\vec{\nabla}T)_k$ and velocity of a fluid induced by motion of the preceding particles in the cluster. In Fig. 1, we show the velocity of the kth particle in the cluster normalized by $2\mu A$.

Now consider the case where a cluster is aligned in the direction normal to the field gradient. It must be noted that in contrast to the previous case, such geometry has a restrictive physical meaning since configuration of the cluster varies during motion in this case. Nevertheless, it allows us to obtain some useful information about the behavior of particles

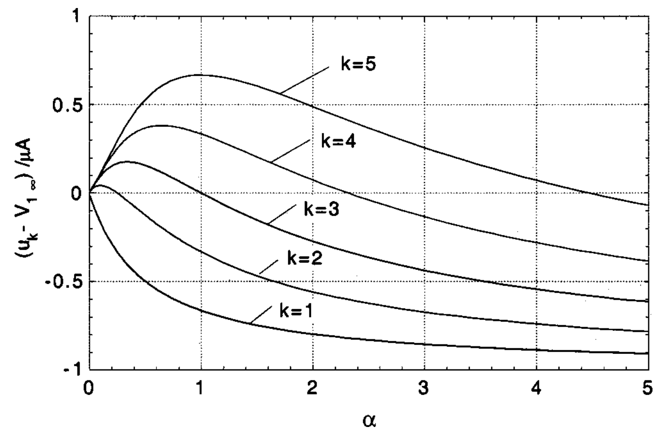

FIG. 2. Dependence of the normalized velocity of the kth particle vs. the ratio of thermal conductivities of the host medium and of the particle $\alpha=\kappa_0/\kappa_1$. Cluster is normal to the external temperature gradient $\vec{A}$.

in gaseous medium with an imposed temperature gradient. The only difference from the previous case is that instead of Eq. (28) we have

$$\vec{V}_{k\infty}=\vec{V}_{k-1,\infty}-\frac{b_{k-1}}{R_{k-1}^3}\vec{A}, \quad \lambda=\lambda_\perp=\frac{3\alpha}{1+2\alpha}.$$

Then, instead of solution (30), we obtain that

$$\vec{V}_{k\infty}-\vec{V}_{1\infty}=\frac{\mu}{3}\frac{3\alpha}{1+2\alpha}\frac{1-\lambda^{k-1}}{1-\lambda}\vec{A},$$

$$\vec{u}_k-\vec{V}_{1\infty}=\frac{\mu}{3}\frac{3\alpha}{1+2\alpha}\left(\frac{1-\lambda^{k-1}}{1-\lambda}-2\lambda^{k-1}\right)\vec{A}.$$

The difference from the previous case (30) is that here the velocity of the flow induced by preceding particles is directed in the opposite direction to the velocity caused by the local temperature gradient $(\vec{\nabla}T)_k$. In Fig. 2, we show the velocity of the kth particle normalized by μA.

The case with stationary particles is the most simple for the experimental investigation of these phenomena. In this case, the particle's velocities $\vec{u}_k=0$ and the velocity field in the vicinity of the kth particle, i.e., for $|\vec{r}-\vec{c}_k|\sim R_k$, can be written similarly to Eqs. (21), (22). Thus, we can present $\vec{V}_k(\vec{r})$ as follows:

$$\vec{V}_k(\vec{r})=\beta_k\vec{A}-\frac{a_k}{|\vec{r}-\vec{c}_k|}[\vec{A}+\vec{n}_k(\vec{A}\cdot\vec{n}_k)] + \frac{b_k}{|\vec{r}-\vec{c}_k|^3}[3\vec{n}_k(\vec{A}\cdot\vec{n}_k)-\vec{A}]. \tag{31}$$

Note that the constant term in formula (31) can be written as $\beta_k\vec{A}$, where coefficient β_k is to be determined, only in two cases, namely, when a cluster is aligned with vector $\vec{A}$ or normal to it.

Hereafter we will assume that, far away from the cluster the gas is at rest so that $\beta_1=0$. Then, using the boundary conditions (23) for the velocity of the first particle, we obtain the known result (see Ref. [8], Chap. 1, Sec. 14),

$$\vec{V}_1(\vec{r}) = -\frac{a_1}{|\vec{r}-\vec{c}_1|}\left[[\vec{A}+\vec{n}_1(\vec{A}\cdot\vec{n}_1)] - \frac{R_1^2}{|\vec{r}-\vec{c}_1|^2} \times [3\vec{n}_1(\vec{A}\cdot\vec{n}_1)-\vec{A}]\right], \tag{32}$$

where $a_1 = -\mu_1 R_1/2$. This velocity implies the force $\vec{F}_1 = 8\pi a_1 \eta \vec{A}$ and a pressure field $p_1 = 2\eta a_1(\vec{A}\cdot\vec{\nabla})1/|\vec{r}-\vec{c}_1| + p_0$. In the region $|\vec{r}-\vec{c}_1| \sim R_1$,

$$\vec{V}_1 = -\mu_1 A \sin\theta_1 \vec{\tau}_1, \quad \vec{\tau}_k = \frac{\partial n_k}{\partial \theta_k}. \tag{33}$$

The total force applied on the particle is determined by two contributions. The first one is due to the pressure gradient induced by preceding particles, and the second one is due to the gas flow caused by an imposed temperature gradient. These two forces can be considered separately, and they depend differently on the particle size. The force due to pressure gradient $\propto R_k^3$ and sharply decreases with the decrease of the particle size. Let us determine the force induced by the gas flow. First consider the case when particles are aligned with the imposed temperature gradient $\vec{A}$. Equation (33) implies that the first particle does not induce a flow in the direction of a temperature gradient. Therefore, the second particle is not subjected to the external flow caused by the first particle, and in Eq. (31) for $k=2$, $\beta_2=0$. The latter implies that $\beta_k=0$ for all k. Thus gas velocity in the vicinity of the kth particle $\vec{V}_k(\vec{r})$ is given by Eq. (32) where index 1 is replaced by k, and a force $\vec{F}_k$ is determined by the following formula:

$$\vec{F}_k = -12\pi\mu\eta \frac{\alpha}{1+2\alpha}\left(\frac{3}{1+2\alpha}\right)^{k-1} R_k \vec{A}. \tag{34}$$

For a given k, the force applied on the particle attains its maximum when $\alpha = 1/2(k-1)$:

$$\vec{F}_{k,\max} = -\vec{F}_0 \frac{3^{k-1}}{2(k-1)}\left(1-\frac{1}{k}\right)^k, \quad \vec{F}_0 = 12\pi\mu\eta\vec{A}R_k. \tag{35}$$

In the limit $k \gg 1$,

$$\vec{F}_{k,\max} = -\vec{F}_0 \frac{3^{k-1}}{2e(k-1)}.$$

Note that in the case of one particle the maximum force is attained when $\alpha \gg 1$, i.e., when thermal conductivity of a host medium κ_0 is much larger than the thermal conductivity of a particle κ_1. However, in a cluster with a large number of particles, the maximum force is applied on the particles with a high thermal conductivity $\alpha \ll 1$. In Fig. 3 we show the dependence of F_k/F_0 vs α.

Now consider a case where stationary particles are aligned in the direction normal to the temperature gradient.

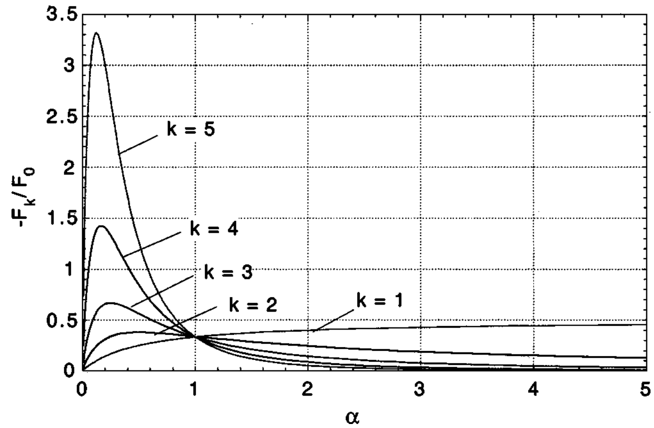

FIG. 3. Dependence of the normalized thermophoretic force acting on the kth particle vs. the ratio of thermal conductivities of the host medium and of the particle $\alpha = \kappa_0/\kappa_1$. Cluster is aligned with the external temperature gradient $\vec{A}$.

In this case, Eq. (33) implies that there exists a flow induced by the first particle $\vec{V}_1 = \mu_1 \vec{A}$. Therefore in this case, apart from the local flow, there exists a flow induced by the preceding particles. Since due to condition (3) $\vec{V}_{k-1}(\vec{r})$ at $|\vec{r}-\vec{c}_{k-1}| \sim R_{k-1}$ is an asymptotic value for $\vec{V}_k(\vec{r})$ at $|\vec{r}-\vec{c}_k| \gg R_k$,

$$\beta_k = \beta_{k-1} - \left(\frac{a_{k-1}}{R_{k-1}} + \frac{b_{k-1}}{R_{k-1}^3}\right). \tag{36}$$

The boundary conditions (23) yield

$$\beta_k - 2\left(\frac{a_k}{R_k} - \frac{b_k}{R_k^3}\right) = 0, \quad \beta_k - \left(\frac{a_k}{R_k} + \frac{b_k}{R_k^3}\right) = \mu_k. \tag{37}$$

Solving Eqs. (36) and (37) we find

$$\beta_k = \mu_{k-1} = \mu\left(\frac{3\alpha}{1+2\alpha}\right)^{k-1},$$

$$a_k = \frac{3}{4}\frac{\mu R_k}{(1+2\alpha)}\left(\frac{3\alpha}{1+2\alpha}\right)^{k-1},$$

$$b_k = a_k \frac{1-4\alpha}{3} R_k^2. \tag{38}$$

The force $\vec{F}_1$ is still determined by Eq. (34) for $k=1$, and forces applied on the other particles in the cluster $\vec{F}_k$ are determined by the following expression:

$$\vec{F}_k = 6\pi\mu\eta \frac{1}{1+2\alpha}\left(\frac{3\alpha}{1+2\alpha}\right)^{k-1} R_k \vec{A}, \quad k \geq 2. \tag{39}$$

For a given k the magnitude of α providing the maximum value of $\vec{F}_k$ is $\alpha = (k-1)/2$ and

$$\vec{F}_{k,\max} = \frac{\vec{F}_0}{2}\left(\frac{3}{2}\right)^{k-1}\frac{1}{(k-1)}\left(1-\frac{1}{k}\right)^k, \tag{40}$$

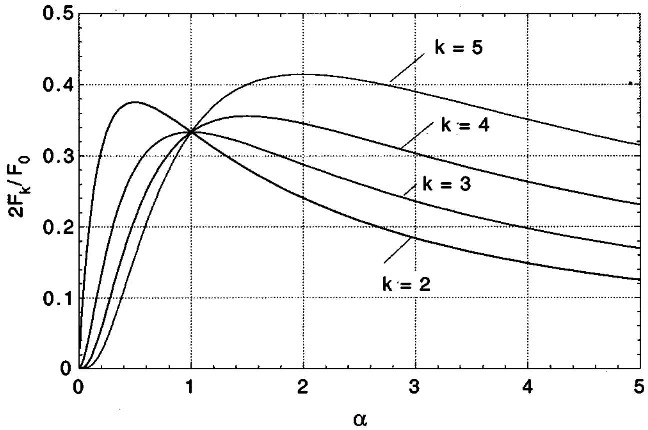

FIG. 4. Dependence of the normalized thermophoretic force acting on the kth particle vs. the ratio of thermal conductivities of the host medium and of the particle $\alpha=\kappa_0/\kappa_1$. Cluster is normal to the external temperature gradient $\vec{A}$.

where $\vec{F}_0$ is determined above. In Fig. 4, we show the dependence of F_k/F_0 vs α. We will not write a general expression for velocity $\vec{V}_k(\vec{r})$, but present expressions for normal and tangential components of the velocity in the basic set $(\vec{n}_k,\vec{\tau}_k)$ normalized by μA_n and μA_τ, respectively:

$$\frac{V_n}{\mu A_n}=\left(\frac{3\alpha}{1+2\alpha}\right)^{k-1}\left[1-\frac{3}{2}\frac{R_k}{|\vec{r}-\vec{c}_k|}\frac{1}{1+2\alpha}\times\left(1-\frac{1-4\alpha}{3}\frac{R_k^2}{|\vec{r}-\vec{c}_k|^2}\right)\right],$$

$$\frac{V_\tau}{\mu A_\tau}=\left(\frac{3\alpha}{1+2\alpha}\right)^{k-1}\left[1-\frac{3}{4}\frac{R_k}{|\vec{r}-\vec{c}_k|}\frac{1}{1+2\alpha}\times\left(1+\frac{1-4\alpha}{3}\frac{R_k^2}{|\vec{r}-\vec{c}_k|^2}\right)\right]. \tag{41}$$

Thus, the magnitude of velocity of gas increases in small scales so that $V\sim[3\alpha/(1+2\alpha)]^{k-1}A$.

V. CONCLUSIONS

In this study, we showed that a linear cluster of particles consisting of N spherical particles embedded in a viscous host medium amplifies various static fields. The simple geometry considered in this investigation allows us to experimentally verify the considered effects. The predicted effects can also be of interest for various environmental and technological applications, e.g., dynamics of atmospheric and combustion aerosols, soot formation, nanotechnology, etc. The obtained results imply the feasibility of separating fine particles from the suspension by injecting into the mixture the particles with the intermediate size.

ACKNOWLEDGMENTS

This study was partially supported by German-Israeli Project Cooperation (DIP) administered by the Federal Ministry of Education and Research (BMBF) and by INTAS (Grant No. 00-0309).

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